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WIND-WAVES IN A LABORATORY TANK: EXPERIMENTS AND THEIR MODELLING

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GENERAL BACKGROUND I

The velocity of wind that excites waves on water surface varies in space and in time, in magnitude as well as in direction. The statistical parameters of wind waves in general also vary in time and in space

Temporal Fourier analysis to calculate frequency spectrum $\hat{\eta}(\omega)$, is **applicable** for **stationary** waves

Spatial Fourier analysis that yields wave vector spectra $\hat{\eta}(\boldsymbol{k})$ is valid only when the wave field is **spatially homogeneous**

Wind waves are usually neither stationary nor homogeneous

The general case is thus too complicated for analysis, simplifications usually are applied:

1. **Duration limited case** – wave field is assumed **homogeneous** and varies in time only, typically under impulsively applied wind forcing;
2. **Fetch-limited case** – **stationary** inhomogeneous wave field under steady forcing evolving in space only

GENERAL BACKGROUND II

Nonlinear theoretical analysis of **broad-banded** water waves is mostly performed for duration-limited approximation in wave-vector Fourier space (**Zakharov equation** for deterministic waves, **kinetic equation** for random waves)

Experiments of Zavadsky & Shemer (JFM 2017) studied waves generated by an impulsively applied wind that necessarily vary **in time AND in space**.

THERE IS NO TEMPORAL EVOLUTION WITHOUT SPATIAL VARIATION
for waves with a preferred propagation direction

Numerical studies of **wave evolution for a duration-limited case** based on theories like the kinetic equation cannot be verified experimentally in a consistent way!

A different theoretical approach is needed!
It should be verifiable in controlled experiments

The simplest realizable in a laboratory facility case that can corroborate the applicability of Fourier-based theoretical nonlinear analysis is the Fetch-Limited Case

FETCH-LIMITED CASE **Steady wind forcing** ($\frac{\partial U}{\partial t}$=0)

All statistical wave parameters at a given location ***x*** are **independent of time *t*** **enabling application of the spatially evolving** **frequency spectrum** $\hat{\eta}(x, \omega)$.

The spatial Fourier analysis yielding wave vector spectra $\hat{\eta}(\boldsymbol{k})$ is apparently inapplicable, existence of the **frequency spectra** allows application of a nonlinear wave evolution model equation

In the vast majority of **laboratory measurements** of wind-waves **steady wind forcing** is applied, but no attempts were made so far to **compare the results with model predictions**

In the present talk, ***experimental results*** *on the wave field evolution with fetch accumulated for a wide range of steady wind forcing conditions* ***compared qualitatively and quantitatively with model simulations***

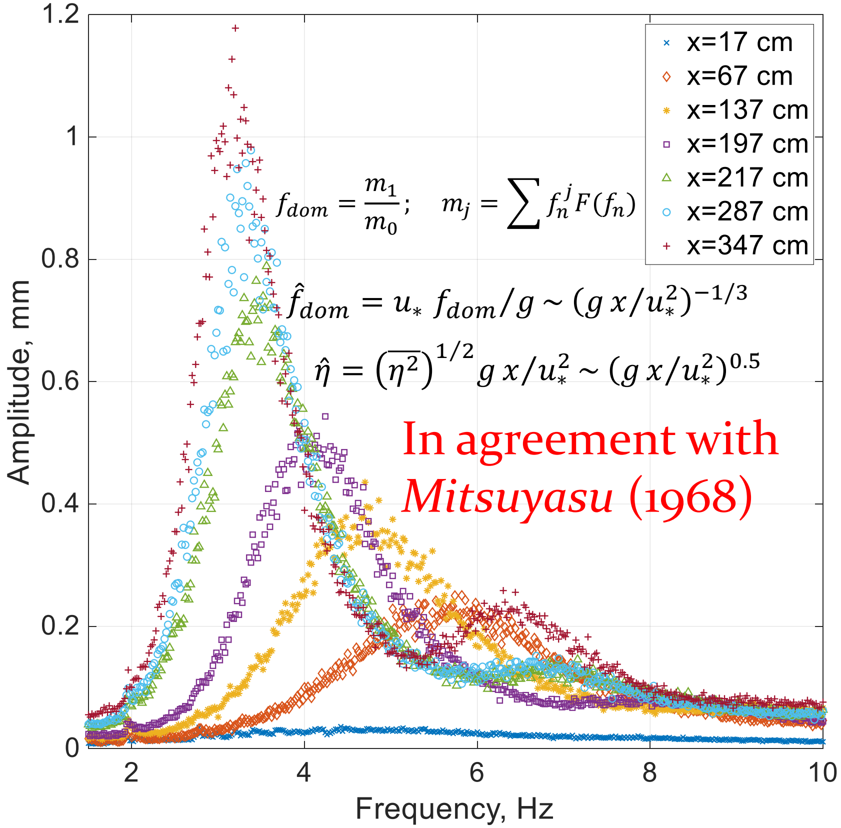
Wave field under steady wind forcing

Results accumulated in our 5 m long wind-wave facility described in *JFM* 2011, *JFM* 2017

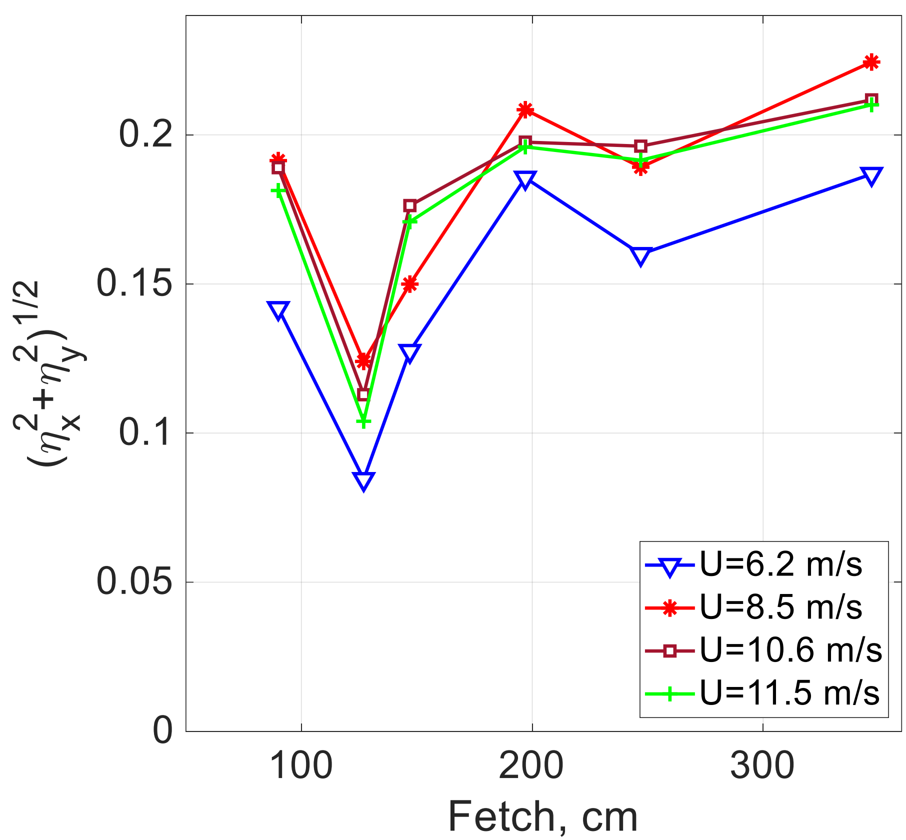
Selected amplitude spectra

U=8.5 m/s, u_*= 0.54 m/s

Wave energy growth and frequency downshifting with fetch



Characteristic wave steepness determined by Laser Slope Gauge (LSG): significant nonlinearity at all fetches and wind velocities.



Summary of the Experimental Findings and Goals

Young fetch-limited wind-waves are random, presumably non-linear, broad-banded, three-dimensional, with peak frequency downshifting and wave energy growth with fetch.

To assess the relative importance of the different contributions, it is imperative to carry out comparison of the experimental results with a suitable theoretical model.

Physical mechanisms that govern wave dissipation and growth due to interaction with wind are not fully understood; there accurate quantitative description is not available yet.

A ***simplified unidirectional model*** *for spatial evolution of wind-waves is suggested. The model is based on experience on deterministic and random waves and on wave breaking gained in our laboratory.*

The simulation results are directly compared with measurements

Basic Assumptions Adopted in the Model

The *semi-deterministic* evolution equation for the variation with fetch x of the amplitude of each frequency harmonic $\hat{\eta}_j = \hat{\eta}(\omega_j, x)$ takes the following shape:

$$c_{g,j} \frac{d\hat{\eta}_j}{dx} = S_{nl} + S_{wind} + S_{diss}$$

The group velocity of the j^{th} harmonic $c_{g,j}$ is applied to relate spatial and temporal variations; the *rhs* represents the temporal rate of change due to nonlinearity, wind input and dissipation.

The measured mean frequency spectrum at a relatively short fetch x_o serves a basis for determination of the initial conditions in the simulations.

To account for wave randomness, multiple realizations of the initial spectrum are obtained by assigning *randomly-distributed phases* to all harmonics. Monte-Carlo simulations of the governing equation are then performed.

The Theoretical Model: The Linear Part

1. Wind input

The wind input is assumed to affect mainly *the vicinity of the local dominant frequency* $f_{\mathrm{dom}}(x)$. The wind input term governing the amplitude of the j^{th} harmonic is modeled as (*Hwang & Sletten* 2008)

$$S_{wind} = \gamma \left(\frac{u_*}{c_{p,dom}}\right)^2 f_{dom}(x)\hat{\eta}_j(x) = \beta_j(x)\,\hat{\eta}_j(x)$$

where the friction velocity u_* is constant for a given wind velocity U, the coefficient $\gamma = a_1$ for $|f_j - f_{dom}(x)| < \Delta f$ and 0 otherwise; $\Delta f = 0.5\, f_{dom}$, in accordance with the typical spectral width in the present measurements.

2. Dissipation

Wave dissipation is mostly associated with breaking, however, for short wind-waves viscous dissipation in the surface boundary layer can also be significant.

Wave breaking is incorporated in the S_{wind} term by adjusting γ. For short water waves, the rate of viscous dissipation $S_{diss} = -2\nu k_j^2 \hat{\eta}_j(x) = \alpha_j \hat{\eta}_j$ (*Lamb, Crapper*). Turbulence is accounted for by effective eddy viscosity ν_{eff}.

The Theoretical Model: II The Nonlinear Part

Nonlinearity is modeled by the *Unidirectional Spatial Zakharov Equation* (*JFM* 2002, *Eur. J. Mech/B-Fluids* 2007)

$$c_{g,j}\, i\frac{dB_j(x)}{dx} = -i \sum_{\omega_j+\omega_l=\omega_m+\omega_n} V_{j,l,m,n} B_l^* B_m B_n e^{-i(k_j+k_l-k_m-k_n)x},$$

The nonlinear interactions kernels $V_{\mathrm{j,l,m,n}}=V(k(\omega_{\mathrm{j}}), k(\omega_{\mathrm{l}}), k(\omega_{\mathrm{m}}), k(\omega_{\mathrm{n}}))$

(*Stiassnie & Shemer JFM* 1984, *Krasitskii JFM* 1994)

Near- resonant quartets: $\omega_j + \omega_l = \omega_m + \omega_n, \ \left|k_j + k_l - k_m - k_n\right| = O(\varepsilon^2 k_{mn})$

The complex wave 'amplitudes' $B_j = B(\omega_j, x)$ are defined by the amplitudes of surface elevation $\hat{\eta}(\omega_j, x)$ and of the surface potential $\hat{\varphi}^s(\omega_j, x)$

$$B(\omega_j, x) = \left(\frac{g}{2\omega_j}\right)^{1/2} \hat{\eta}(\omega_j, x) + i\left(\frac{\omega_j}{2g}\right)^{1/2} \hat{\varphi}^s(\omega_j, x)$$

The **FULL SPATIAL EVOLUTION EQUATION** for 'amplitudes' $B_j = B(\omega_j, x)$ is thus

$$c_{g,j}\frac{dB_j(x)}{dx} = -i \sum_{\omega_j+\omega_l=\omega_m+\omega_n} V_{j,l,m,n} B_l^* B_m B_n e^{-i(k_j+k_l-k_m-k_n)x} + \beta_j B_j(x) - \alpha_j B_j(x)$$

Nonlinearity — **Wind input** — **Dissipation**

The linear approximation

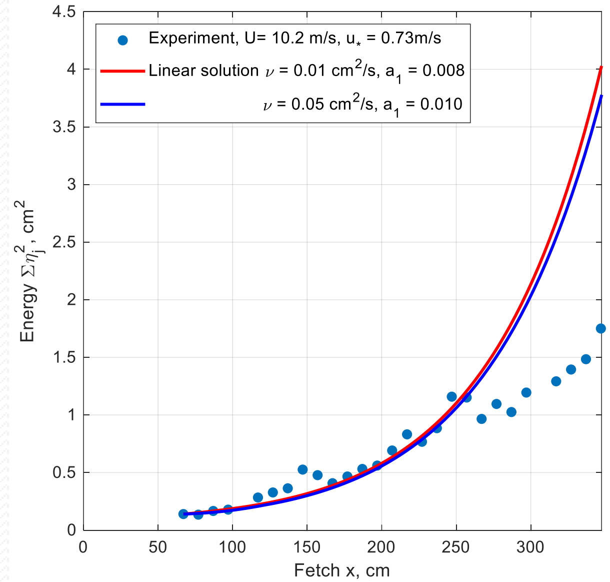
Simulations *vs.* Experiments

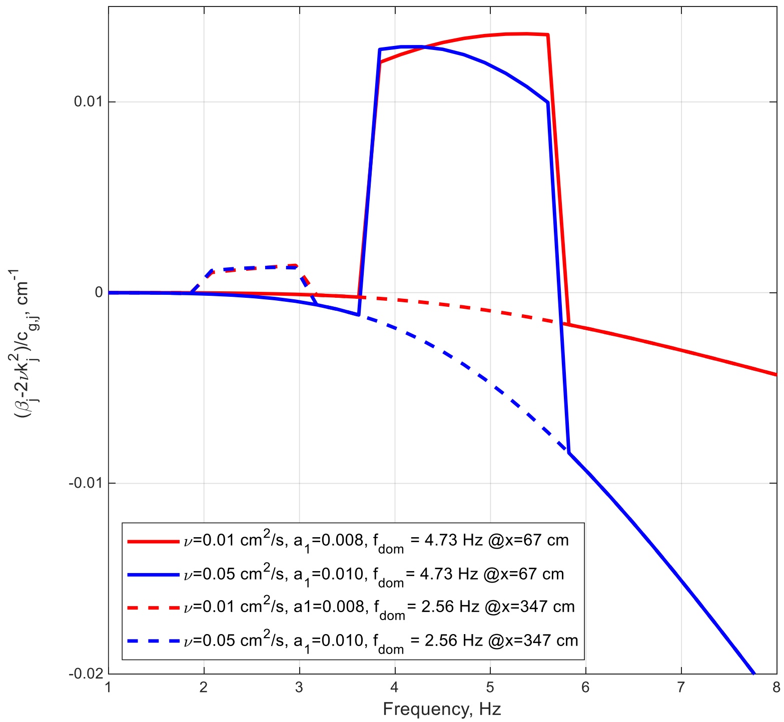
$$c_{g,j}\frac{dB_j(x)}{dx} = -i \sum_{\omega_j+\omega_l=\omega_m+\omega_n} V_{j,l,m,n} B_l^* B_m B_n e^{-i(k_j+k_l-k_m-k_n)x} + \beta_j B_j(x) - \alpha_j B_j(x)$$

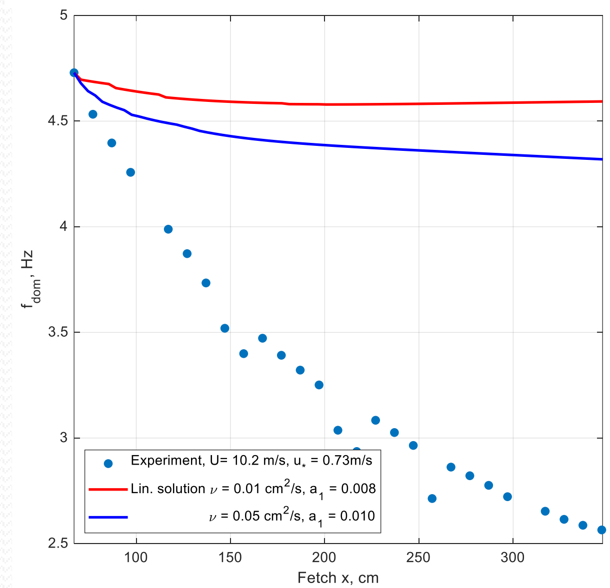
Linear solution: $B_j = B(f_j, x) = B(f_j, x_0)\,exp\left[\frac{(\beta_j(x)-\alpha_j)(x-x_0)}{c_{g,j}(x)}\right]$

Linear growth/decay exponent U=10.6 m/s, u_*=0.73 m/s

The dominant frequency f_{dom} and thus β_j are **adjusted** every **half dominant wave length $\lambda_{dom}/2$**



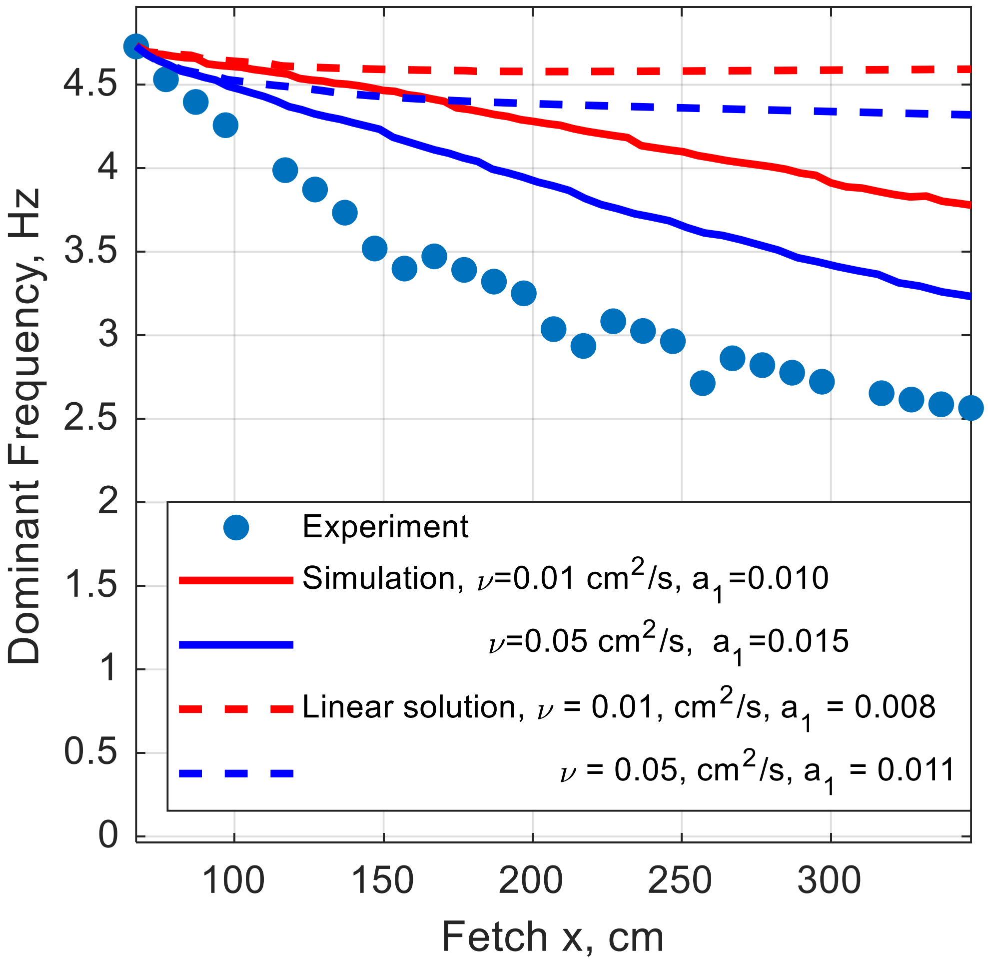




Dominant Frequency in nonlinear Monte-Carlo Simulations

Comparison with the linear solution and measurements

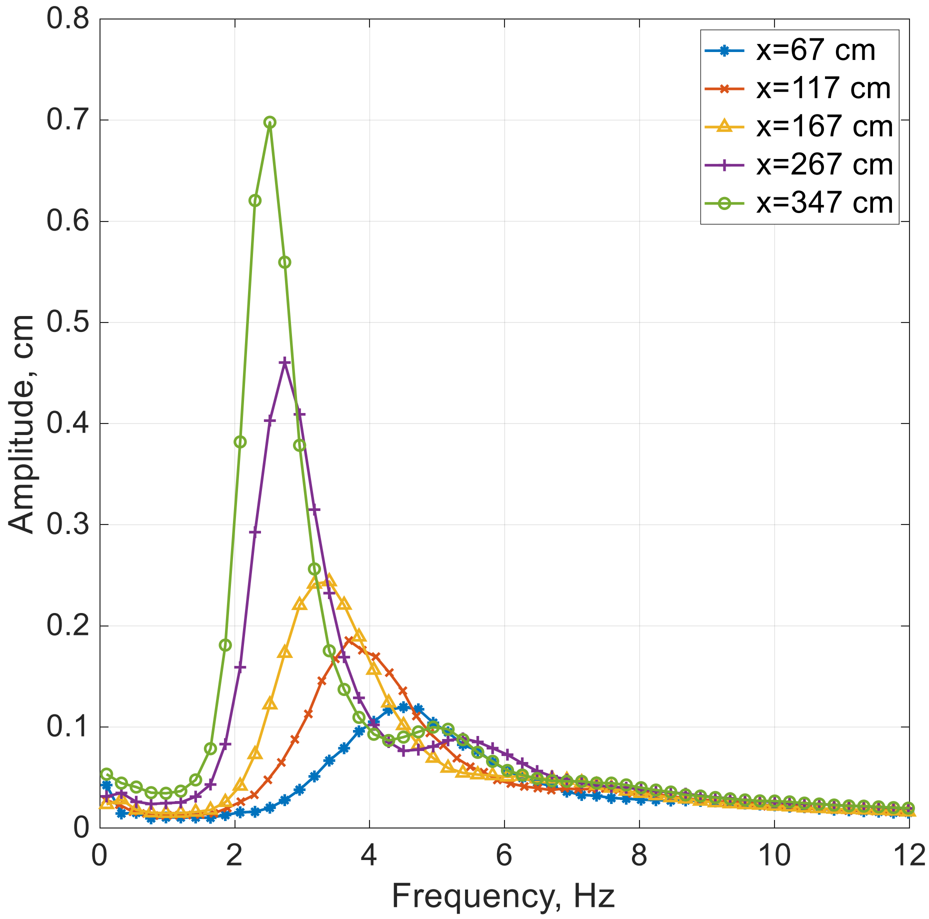
Wind velocity U =10.62 m/s; random initial phases; average of 100 realizations



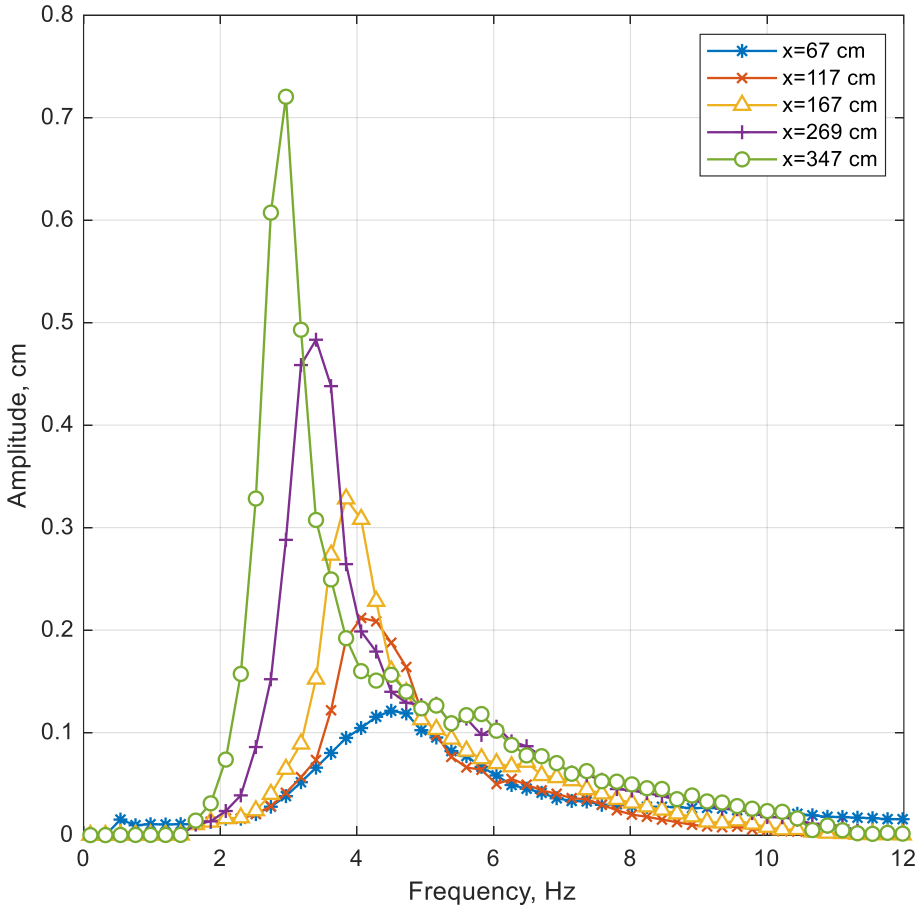
Evolution of wave spectra along the tank:
Experiment vs. Monte Carlo simulations

based on random initial phases and f_{dom} adjustment each $\lambda_{dom}/2$

Experiment



Simulations



Conclusions

- **DIRECT COMPARISON** of numerical simulations with experiments demonstrates qualitative and quantitative agreement
- Simplified unidirectional model allows studying spatial evolution of young broad-banded wind-waves along the tank under steady wind forcing.
- The model accounts for the essential spatial inhomogeneity of an evolving wind-wave field that renders nonlinear models based on wave vector spectra inapplicable.
- The model allows decoupling effects of wind input, dissipation and nonlinearity; it demonstrates the relative importance of nonlinearity in frequency downshifting.
- While the essential contribution of nonlinearity is clearly demonstrated, linear modeling of wind input and of dissipation can grasp some important features of waves development with fetch. The linear part may be more prominent for less steep older waves
- The modular character of the model allows incorporation of alternative possible mechanisms for wind input and wave energy dissipation


Thank you

The suggested simplified model takes advantage of the experience accumulated in our laboratory in the theoretical studies of nonlinear wave evolution, wave tank experiments on deterministic and random waves, extensive experiments on wind-wave evolution and of wave breaking under controlled conditions

$$f_{dom} = \frac{m_1}{m_0}; \quad where \quad m_j = \int_{f_{\min}}^{f_{\max}} f^{\,j} F(f)\,df.$$

Dimensionless spectral width

$$\nu = \sqrt{\frac{m_0 m_2}{m_1^2} - 1}$$

The equation accounts for 4-wave (quartet) wave-wave interactions in a **deterministic** and **conservative** wave system

Modifications are required to account for:

1. **Wind input**
2. **Wave energy dissipation**
3. **The stochastic and 3D nature of wind waves**

Initial conditions should be prescribed matching those in the experiments

TWO COMMON SIMPLIFIED CASES

I. DURATION-LIMITED CASE:

Spatially homogeneous wave field is considered – no dependence on fetch

All statistical parameters depend **ON TIME ONLY**

Duration-limited case was considered in all groundbreaking studies (Jeffreys, Miles, Phillips, Valenzuela, Kawai, etc.)

Since **wind** always **has a preferred direction**, wind-waves are never spatially homogeneous, and the **duration-limited case cannot be realized** in practice

Temporal evolution of spatially uniform wave field is under impulsive wind forcing is usually considered in the theoretical analysis.

ENERGY BALANCE OF WIND-GENERATED WAVES

$$\frac{DE(\omega)}{Dt} = \frac{\partial E(\omega)}{\partial t} + (\boldsymbol{c_g} \cdot \boldsymbol{\nabla_x})\, E(\omega) = Q_{nl} + Q_{in} + Q_{dis}\ ;$$

where the horizontal gradient operator $\boldsymbol{\nabla_x} = \hat{\boldsymbol{\imath}} \frac{\partial}{\partial x} + \hat{\boldsymbol{\jmath}} \frac{\partial}{\partial y}$

$\boldsymbol{c_g} = \boldsymbol{c_g}(\omega)$ is the group velocity vector for the free frequency harmonic ω

For a fetch-limited case,

wave spectral **energy density** $E(\omega)$,

nonlinear interactions rate $Q_{nl}(\omega)$

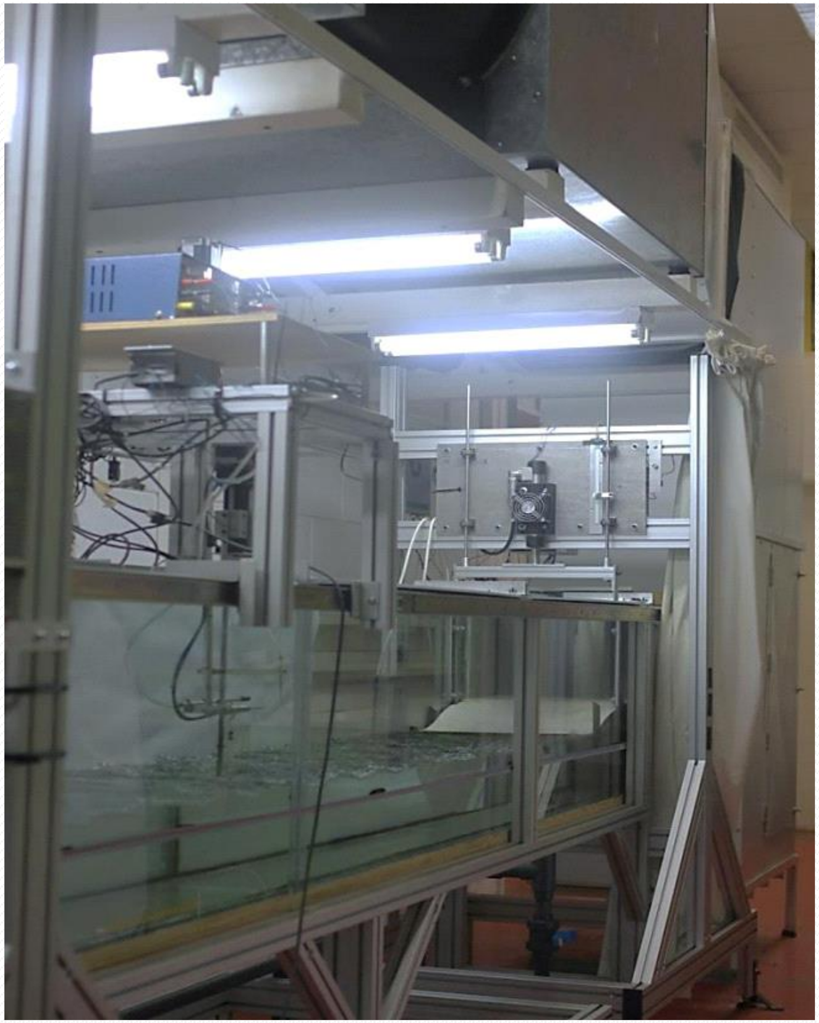
wind **input** $Q_{in}(\omega)$

wave **energy dissipation rate** $Q_{dis}(\omega)$

depend on **space coordinate** $\boldsymbol{x}$

To predict evolution of the statistical parameters with fetch $\boldsymbol{x}$, the terms $Q_{nl}(\omega)$, $Q_{in}(\omega)$ and $Q_{dis}(\omega)$ have to be evaluated

TAU Facility for Study of Young Wind Waves



Length 5 m, Width 0.4 m
Height 0.5 m (water depth ≈0.2 m)

Programmable Blower
Maximum wind speed U ≈15 m/s

Surface elevation measured by capacitance-type wave gauges
Instantaneous surface slope components $\partial\eta/\partial x$ and $\partial\eta/\partial y$ by a Laser Slope Gauge

Wind generation, sensor position and calibration, as well as data acquisition are fully automatic controlled by LabView

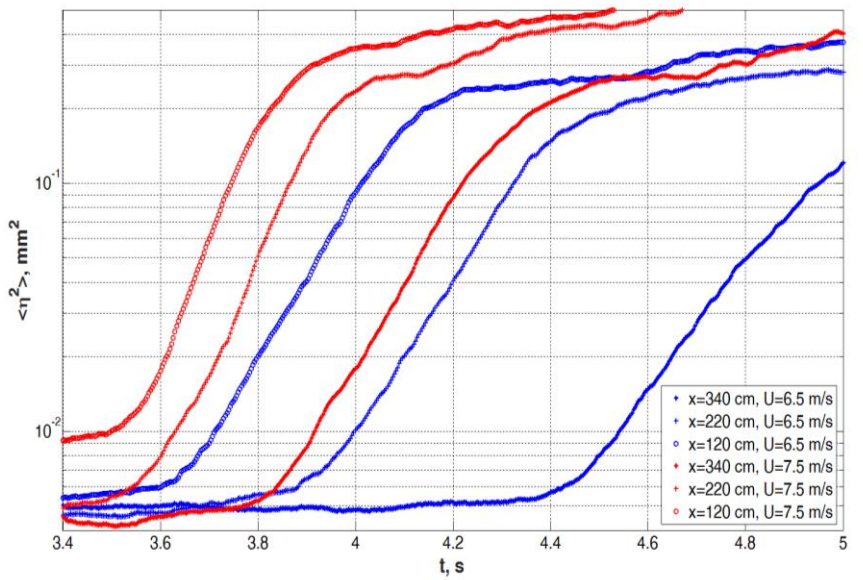
Experimental facility and procedure are described in detail in
Zavadsky & Shemer, *Journal of Visualized Experiments*, e56480, 2018

LINEAR APPROACH TO WIND-WAVES GROWTH

Temporal Growth

$\beta = \frac{1}{E}\frac{\partial E}{\partial t}$ - temporal growth rate

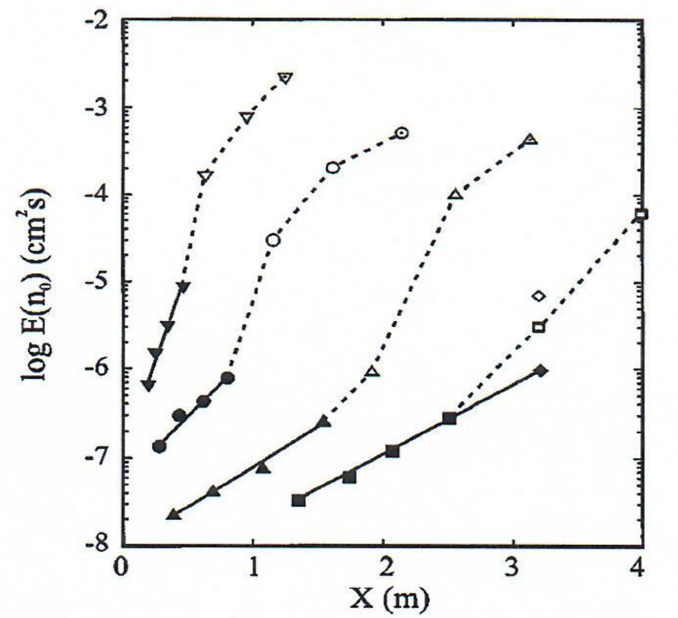
Zavadsky & Shemer *JFM* 2017



β, s⁻¹				
X, cm	U = 6.5 m/s	U = 7.5 m/s	U = 9.5 m/s	U = 10.5 m/s
120	7.6	12	11.7	12.2
220	7.1	9.4	12	13.9
340	5.9	7.8	10	10.7

Spatial Growth $\gamma = \frac{1}{E}\frac{\partial E}{\partial x}$

Caulliez et al. *Phys. Fl.* 1998

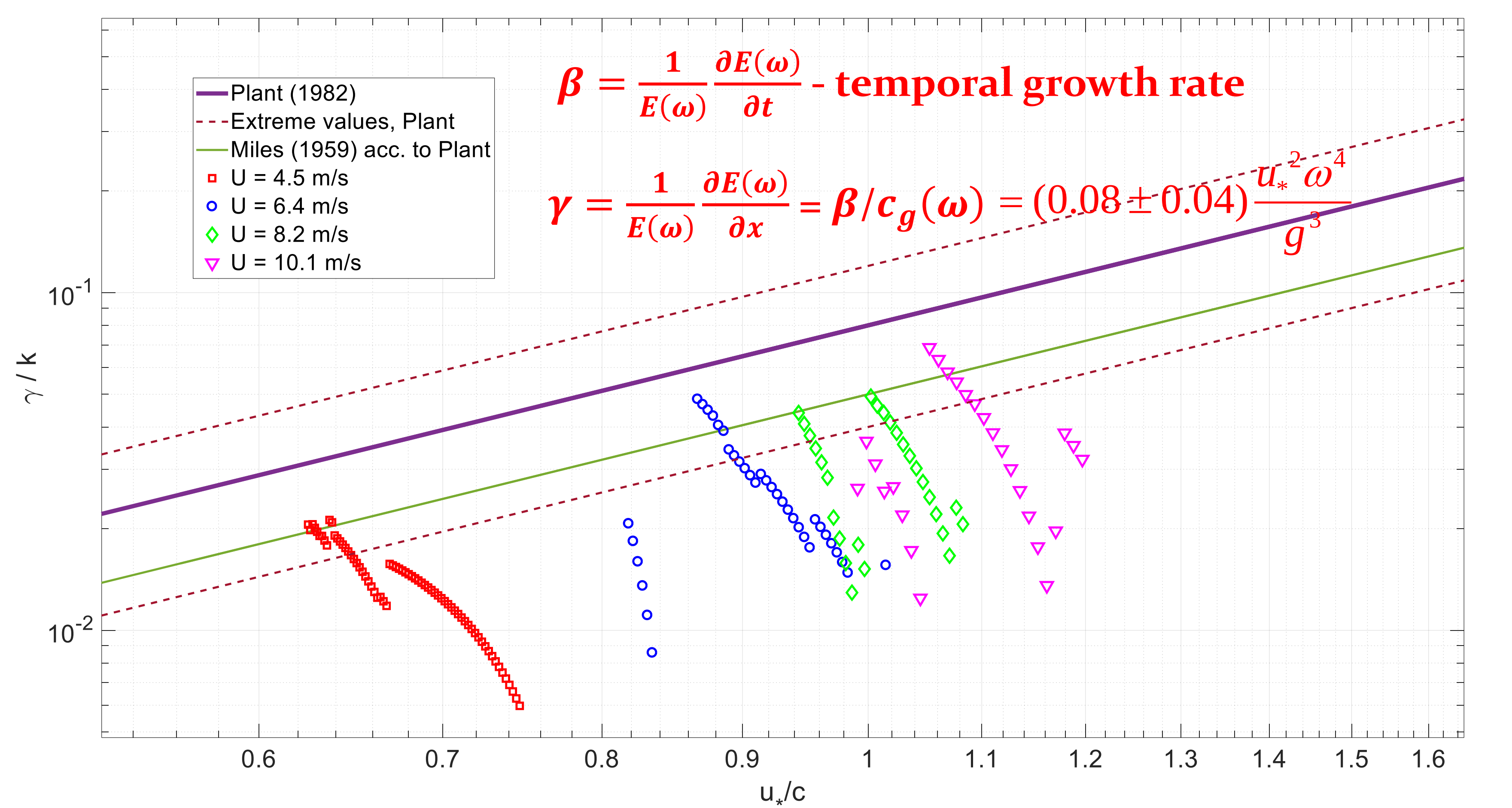


These results show variation of the **total wave energy *E*** rather than specific **frequency component $E(\omega)$**

Spatial exponential growth rates for various constant wind velocities

Symbols –measurements in our wind-wave facility, lines - Plant (*JGR* 1982)

Liberzon & Shemer, *JFM 2011*



Variation along the test section of the LSG-measured characteristic surface slope values for two wind velocities *U*